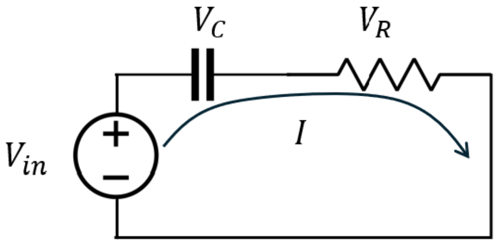
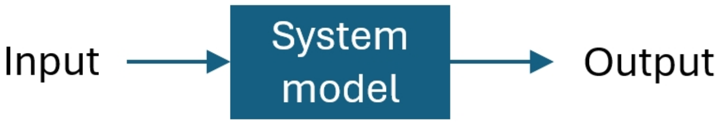
Task 1 – System Modeling



In this lab, we will use a series RC circuit as an example to study the first order system. The key components of the system are:

- System input – voltage of the power source, V_{in}
- System output – voltage of the capacitor , V_C
- System model



For a series RC circuit, we have,

$$\text{Input: } V_{in}(t) = V_R(t) + V_C(t) \tag{3.1}$$
$$= I(t)R + +V_C(t) \tag{3.2}$$
$$\text{Output: } V_C(t) = \frac{1}{C}\int i(t)dt \tag{3.3}$$

Transforming to the Laplace domain, we will have:

$$\text{Input: } V_{in}(s) = I(s)R + V_C(s) \tag{3.4}$$
$$\text{Output: } V_C(s) = \frac{1}{C}\frac{1}{s}I(s) \tag{3.5}$$

Report Item 1-a

Derive the transfer function step-by-step. In previous equations, the $V_{in}(s)$, $V_C(s)$ and $I(s)$ are variables, C and R are constants.
Type down your final transfer function in standard form (coefficient of the highest-order term in the denominator equal to 1.)

$$\frac{\text{Output}}{\text{Input}} = \frac{V_C(s)}{V_{in}(s)} = ???$$

In this lab, we will study how such system responds to a step input signal.

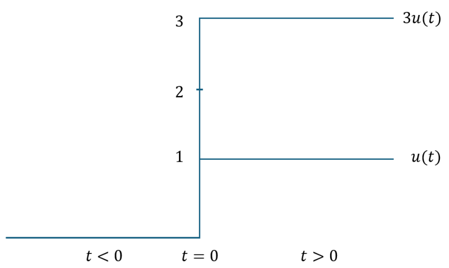
If we use a unit step signal $u(t)$ as input, its Laplace transform is,

$$V_{in}(s) = \mathcal{L}(u(t)) = \frac{1}{s}$$

Report Item 1-b

Consider a step signal with an amplitude A, written as $Au(t)$. The ampltide A is a constant here. Its Laplace transform will be

$$V_{in}(s) = \mathcal{L}(Au(t)) = ???$$



An example of i) a unit step, $u(t)$, ii) a step with amplitude of 3, written as $3u(t)$

Now, given the transfer function of the system $\dfrac{V_C(s)}{V_{in}(s)}$ in Item 1-a and the input signal $V_{in}(s)$ in Item 1-b,

Report Item 1-c

The output of the system in Laplace will be

$$V_C(s) = \frac{V_C(s)}{V_{in}(s)} \times V_{in}(s) = ???$$

Perform Partial Fraction, the output of the system in Laplace can be expressed as

$$V_C(s) = ???$$

Perform inverse Laplace transform. The time-domain output will be

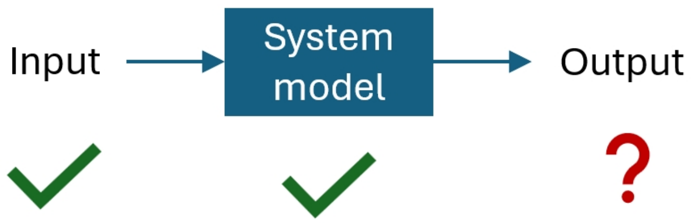
$$V_C(t) = \mathcal{L}^{-1}(V_C(s)) = ???$$

Task 2 – Study the System Output

In this task, we assume:

- System input is given
- System output is unknown
- System model is given

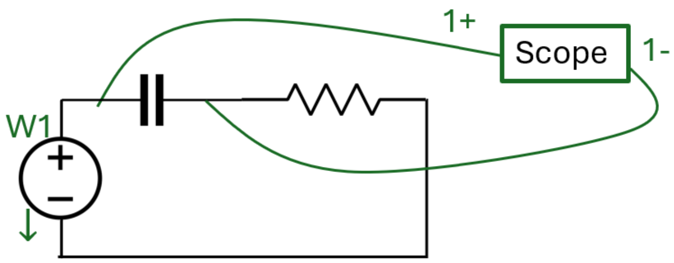
We want to check if the experimental output of the circuit matches the theoretical response obtained from the mathematical model.



Task 2.1 – Wire the circuit

We build a series RC circuit using:

- 100 kΩ resistor
- 0.22 μF capacitor
- Analog Discovery source (W1 pin Source, ↓ pin Ground)



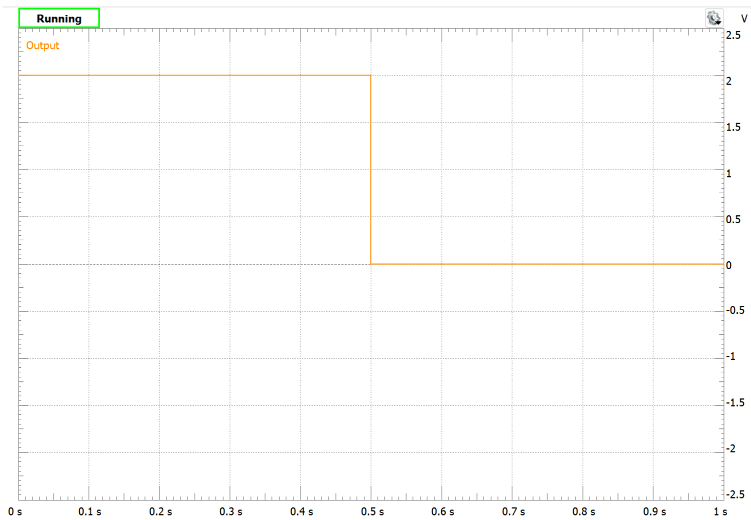
** Green color indicates connection to Analog Discovery

Task 2.2 – Capture step response

Firstly, we set up the step signal as $V_{in}(t)$ in the Waveforms softwave:

Type = Pulse	Frequency = 1 Hz
Amplitude =??	Offset = ??
Symmetry = 50%	Phase = 0

You need to determine the amplitude and offset so that the signal ranges between 0 and 2 volts.



Remember to **Run** the Scope window to actually sending out the signal.

Secondly, use the scope of Analog Discovery to capture the output signal $V_C(t)$. Adjust the scope settings to get a good-looking display.

Let the scope only show a **single-cycle display** for better data analysis.

Potential Troubleshooting

- Alway Fix "Time → Position" to be "0".
- If you want to zoom in/out the Voltage-axis (Y-axis), adjust "Channel → Range".
- If you want to move signal up/down over the Voltage-axis (Y-axis), adjust "Channel → Offset"
- If you want to zoom in/out the Time-axis (X-axis), adjust "Time → Base".
- If the signal keeps scrolling on the display, adjust triggering level voltage.

Task 2.3 – Measure the time constant

We stay with the scope in the Waveforms softwave. Now, we will measure a special thing known as **Time Constant**.

To understand what a time constant is, let's revisit Task 1. In Item 1-c, your final result might look like:

$$Output = A_{ss}(u(t) - e^{-\frac{1}{RC}t}) \tag{3.6}$$

Where R is the resistance, C is the capacitance. The A_{ss} here is the Steady-State Amplitude.

If you plug in $t = \infty$, the output becomes:

$$Output = A_{ss} \times (1 - e^{-\infty}) = A_{ss} \tag{3.7}$$

If you plug in $t = RC$, the output becomes:

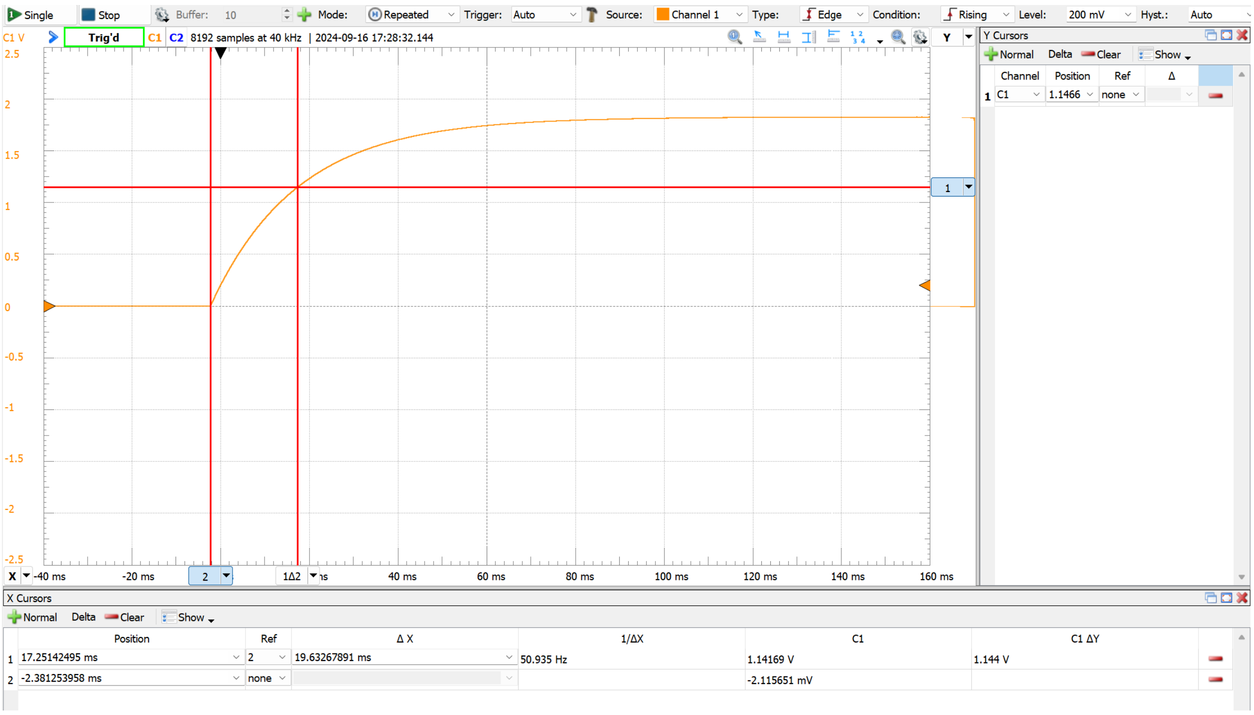
$$Output = A_{ss} \times (1 - e^{-1}) \approx 0.632 A_{ss} \tag{3.8}$$

That is,
for a first-order system, regardless of the amplitude of the step input, the output will always pass through a specific point of $0.632 \times$ Steady-State Amplitude at a specific time.

This specific time is called **Time Constant**. Here in our case, this time is $t = RC$. The time Constant only depends on of parameters of the system itsel. The time Constant has nothing to do with the input or output.

So, how to measure the Time Constant on a scope? We need to follow these steps:

- Step 1: Use cursors to measure the Steady-State amplitude A_{ss} on the scope.
- Step 2: Calculate the value $= 0.632 \times A_{ss}$
- Step 3: Use cursor(s) on the scope to locate the point on the signal where the amplitude is equal to $0.632 \times A_{ss}$
- Step 4: Once you have identified this point, use the cursors to measure the **horizontal distance** from the start of the cycle to this specific point. This distance is the time constant.



Cursors for Step 3 and Step 4. Step 3 uses a Y cursor to look for the specific point. Step 4 uses two X cursors to measure the horizontal distance. Two X cursors need to set up a "Ref" for distance ΔX measurement.

Report Item 2-a

Take a screenshot of your scope display. Make sure to include the cursors and readings in the picture.

Make sure that the local time and device Serial Number are included.

Report Item 2-b

Fill the table.

The first 3 entries are based on the scope measurement.
The last 2 entry compares with the theoretical time constant, which equals to RC.

Steady-state Amplitude A_{ss}	??
$0.632A_{ss}$	??
Experimental Time Constant	??
Theoretical Time Constant	??
Percent error	??

Check Point 1 – Scope display and table

Task 2.4 – Data visualization (Individual)

Save the scope data to a .csv file.

Report Item 2-c

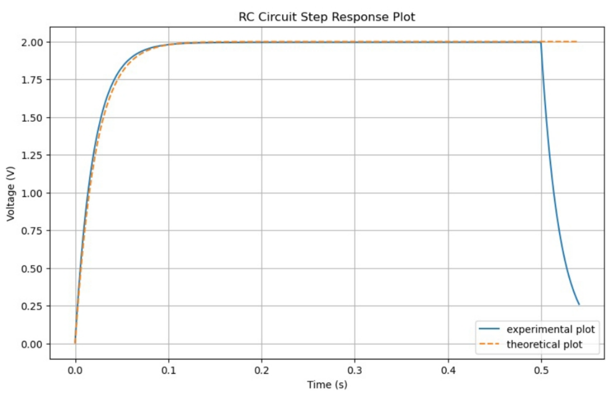
In Jupyter Notebook, use Python code the plot on the same figure:
1) the experimental output signal,
2) the theoretical output signal (1-c result with R, C, A values plugged in).
You need to show both code and generated graph in the report.

* Hint1: In Python Pandas, the method `.iloc[IndexA : IndexB]` selects rows from IndexA to IndexB.

* Hint2: In Python Matplotlib, if you change the function `plt.plot(t,y_t)` to `plt.plot(t+2,y_t)` . The $y(t)$ signal will shift to right over the Time-axis by 2 seconds

* Feel free with expore the solutions with ChatGPT.

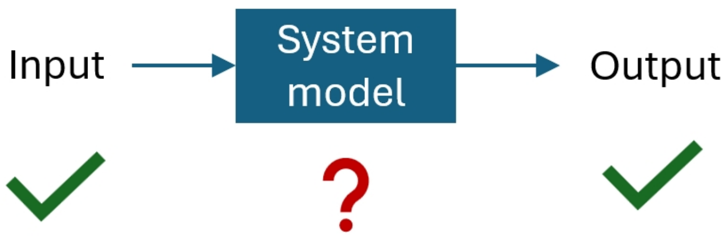
Here is my example plot:



Task 3 – System Identification

In this task, we assume:

- System input is given
- System output is given
- System model is partially unknown



In the task, we assume the capacitor value is unknown, while the rest of transfer function is known.

Task 3.1 – Data collection

In the lab, we continue to use the "unknown" device box. Recall the box connection:

- Orange Port: Configures the box as an unknown capacitor.
- Blue Port: Configures the box as an unknown inductor.
- Black Port: Output port.

In this lab we will check the unknown capacitor.

In the RC circuit, replace the original capacitor with the unknown device box.

We will collect the data of 3 input-output pairs to estimate the capacitor. Generate step input signals with 3 amplitudes: 2 volts, 1 volt, and 0.5 volts. Measure the corresponding time constants from the scope.

Report Item 3-a

Fill the table. The columns 1-3 are for output on the scope. The column 4 is based on calculation.

Steady-state amplitude A_{ss}	$0.632A_{ss}$	Time constant	Calculated capacitance

Potential Troubleshooting

If the signal hasn't reached steady state before switching to the next cycle, your RC circuit have a large time constant. You will need to increase the time period of the input signal, so that the scope can fully display one cycle.

Check Point 2 –Table and averaged capacitance

Also obtain the ground truth value from the lab staff.

Report Item 3-b

Based on the table, the average of the 3 calculated capacitance is ???.
State the box ID and the ground truth value.
Is your calculated value close to the ground truth value?

Lab Report

Please complete the report individually.

Write the report in Jupyter Notebook. Then export it as .html. Finally convert .html to .pdf. The submission in BlackBoard must be in .pdf.

The total score for the report is 20 points.

- Format. (3 points)
- Item 1-a, 1-b, (2x1 points) , 1-c (2 points)
- Item 2-a, 2-b, 2-c (3x2 points)
- Item 3-a, 3-b (2x2 points)
- Conclusion. (3 points)